Example Compute the length of the curve $(x,y) = (t^3 - t, 2 - t^2 + 3t)$ for $0 \le t \le 1$.

Solution: $L = \int_0^1 \sqrt{(x')^2 + (y')^2}\, dt$

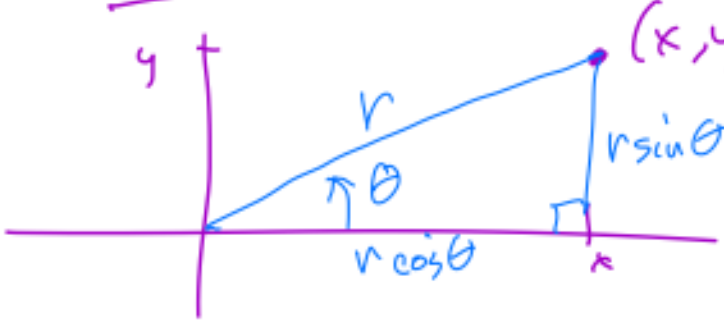
$= \int_0^1 \sqrt{(3t^2-1)^2 + (-2t+3)^2}\, dt$

Type some Sage code below and press Evaluate.

```
1  var('t')
2  xx(t)=t^3-t
3  yy(t)=2-t^2+3*t
4  pp=parametric_plot((xx(t),yy(t)),(t,0,1))
5  show(pp)
6  show(n(integral(sqrt(xx(t)^2+yy(t)^2),t,0,1)))
```

Evaluate

Calculus in Polar Coordinates



$(x,y) = (r,\theta)$ in polar coords.

$x = r\cos\theta$

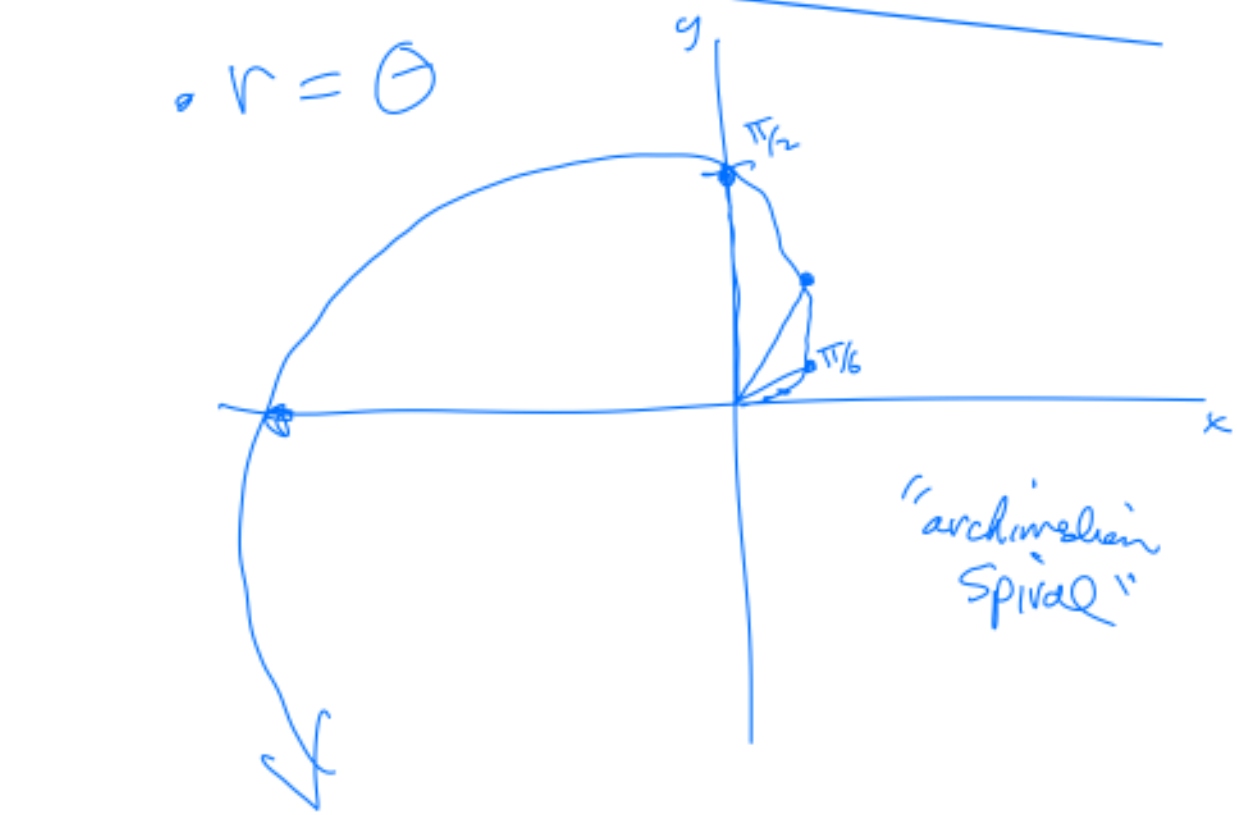
$y = r\sin\theta$.

Some interesting polar graphs:

r = function of θ

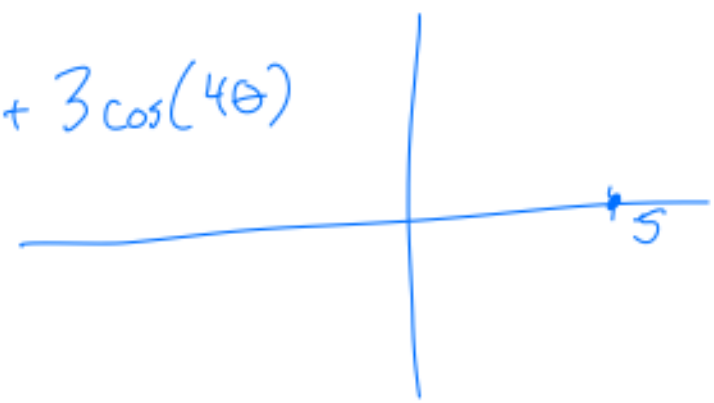
Examples:

- $r = \theta$



"archimedian spiral"

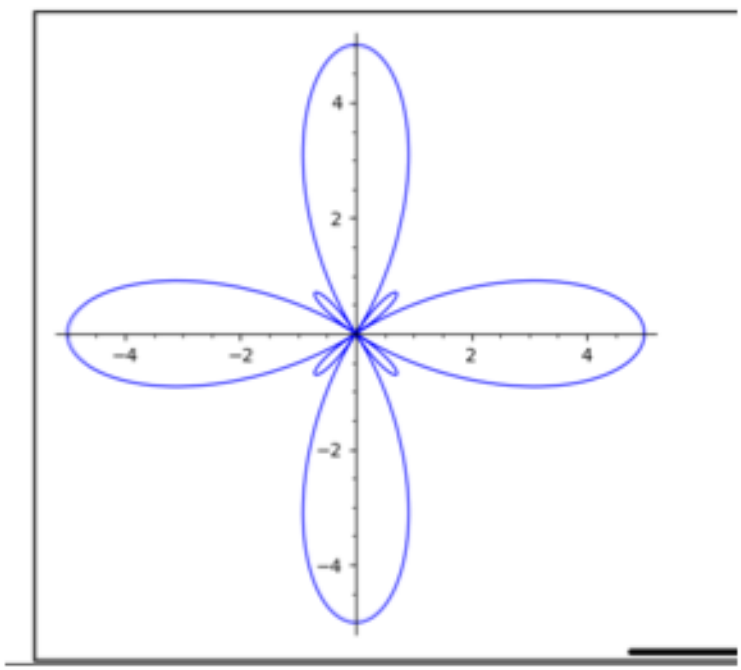
- $r = 2 + 3\cos(4\theta)$



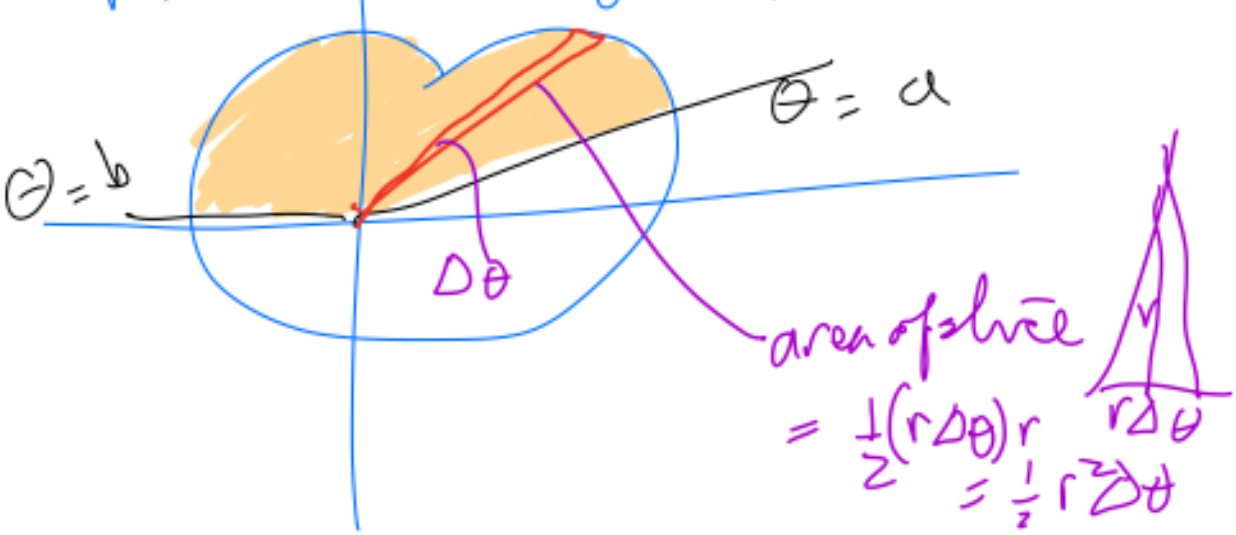
Type some Sage code below and press Evaluate.

```
var('t')
f(t)= 2+3*cos(4*t)
polar_plot(f(t),t,0,2*pi)
```

Evaluate



Find the area of a polar plot.



$$\therefore \text{Area} = \frac{1}{2}\int_{\alpha}^{b} r^2\, d\theta$$

Example: find area of upper part of $r = 1 + \cos\theta$.

$\theta = \pi$ $\theta = 0$

$$\text{area} = \frac{1}{2}\int_0^{\pi} (1+\cos\theta)^2\, d\theta$$

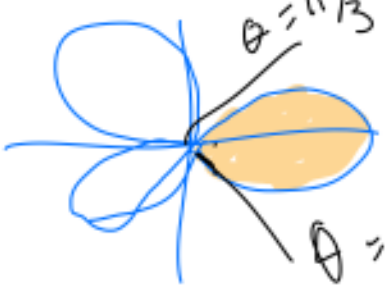
$$= \frac{1}{2}\int_0^{\pi} 1 + 2\cos\theta + \underbrace{\cos^2\theta}_{\frac{1}{2}+\frac{1}{2}\cos(2\theta)}\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos(2\theta)\right) d\theta$$

(the $2\cos\theta$ and $\frac{1}{2}\cos(2\theta)$ terms integrate to 0)

$$= \boxed{\frac{3}{4}\pi}$$

Example – Find area of one loop of $r = 1 + \cos(3\theta)$



$$\text{Area} = \frac{1}{2}\int_{-\pi/3}^{\pi/3} (1+\cos(3\theta))^2\, d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3} \left(1 + \underbrace{2\cos(3\theta)}_{0} + \underbrace{\cos^2(3\theta)}_{\frac{1}{2}+\frac{1}{2}\cos(6\theta)}\right) d\theta$$

(the $\frac{1}{2}\cos(6\theta)$ term integrates to 0)

$$= \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{2\pi}{3} = \boxed{\pi/2}$$

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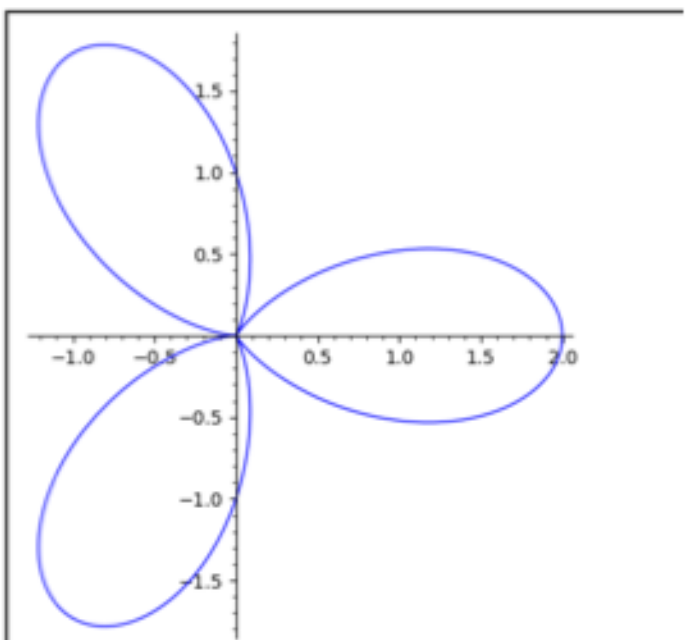
Type some Sage code below and press Evaluate.

```
1  f(t)=1+cos(3*t)
2  show(polar_plot(f(t),t,0,2*pi))
3  show(1/2*integral(f(t)^2,t,-pi/3,pi/3))
```

Evaluate



$$\frac{1}{2}\,\pi$$